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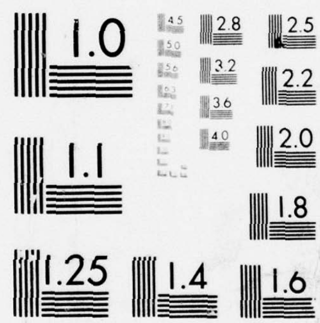

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A NONSTANDARD THEORY OF GAMES. PART II.

ON NON-ATOMIC REPRESENTATIONS OF *FINITE GAMES

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10 Alain A. Lewis*

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Introduction

The idea of using Loeb's construction to obtain measurable representations of *Finite games we owe to Professor Don Brown of Yale University and remark that Professor Salim Rashid of Dartmouth College first obtained results analogous to Theorem I.13 for the case of the core of a nonstandard exchange economy in his dissertation at Yale written under Professor Brown (Reference 10). An additional framework of application for Loeb's construction has been obtained by Rashid along with Professor Robert Anderson of Princeton University in a nonstandard characterization of weak convergence (Reference 11). Also, the work of Professor H. Jerome Keisler in "Hyperfinite Model Theory," Logic Colloquium 1978 (North Holland Press, 1979), and "An Infinitesimal Approach to Stochastic Analysis," Preliminary Paper, University of Wisconsin, 1978, employs Loeb's construction to elegantly derive results concerning random social phenomena and the mathematics of stochastic processes.

I. Loeb Space Representations

We demonstrate that a *Finite cooperative game, $\Gamma^* = \langle F^*, A(F^*), v^* \rangle$, when viewed as a construction on a nonstandard *Finite measure space of participants $\Phi^* = \langle F^*, A(F^*), u_{F^*} \rangle$, where u_{F^*} is *Finitely additive, has a standard non-atomic representation, $\psi(\Gamma^*)$, on $\psi = \langle X(F^*), X(A(F^*)), m \rangle$, where $X(A(F^*))$ is the smallest σ-algebra containing $A(F^*)$. Further, the set of u_{F^*}-measurable payoff configurations (x^*, F^*) of Γ^* for $u_{F^*}(S) = \frac{\|F^* \cap S\|}{\|F^*\|}$ and $S \in A(F^*)$, are such that $st(x^*)$ is m-measurable in $\psi(\Gamma^*)$, and such that

$$\left[\frac{1}{\|F^*\|} \sum_{j \in F^*} x^*(j) = \int_{X(F^*)} st(x^*)\, dm\right]_{\text{Mod } M_1}$$

Definition I.1: Consider a nonstandard *Finite measure space, $\Phi^* = \langle A(F^*), u_{F^*} \rangle$ in the sense of [1], [2], or [3], where $F^* = [0, \omega]$, $\omega \in N^* - N$, $A(F^*)$ is the internal algebra of sets in F^*, and $u_{F^*}(S) = \frac{\|F^* \cap S\|}{\|F^*\|}$ for $S \in A(F^*)$.

A *Finite cooperative game on Φ^* is a triplet $\Gamma^* = \langle F^*, A(F^*), v^* \rangle$ for v^* superadditive on $A(F^*)$ and $v^*(F^*)$ Q-bounded in R^*_+ with $v^*(\phi) = 0$.

The set of payoff configurations for Γ^* is denoted as (x^*, F^*), where $x^* : F^* \to R^*_+$, and for every player $j \in F^*$, $x^*(j) \in M_0^+$ such that

$$\left(\sum_{j \varepsilon F^*} x^*(j) = v^*(F^*) \right)_{\text{Mod } M_1}$$

The Q-boundedness of $v^*(F^*)$ allows normalization by $\frac{1}{\|F^*\|}$, so that the set of payoffs for Γ^* then satisfies the condition that

$$\left(\frac{1}{\|F^*\|} \sum_{j \varepsilon F^*} x^*(j) = \tilde{v}^*(F^*) \right)_{\text{Mod } M_1}$$

where $\tilde{v}^*(F^*) = \frac{1}{\|F^*\|} v^*(F^*)$. Alternatively, the set of payoff configurations can be characterized as:

$$(x^*,F^*) = \left\{ x^* \varepsilon (R^*_+)^{F^*} : (\forall j \varepsilon F^*) x^*(j) \varepsilon M_0^+ \ldots \left(\int_{F^*} x^* du_{F^*} = \tilde{v}(F^*) \right)_{\text{Mod } M_1} \right\}$$

by definition 4.1 of [1].

Definition I.2: A function $f : F^* \to R^*$ is said to be u_{F^*}-measurable if:

$$\left(u_{F^*}(S) = 0\right)_{\text{Mod } M_1} \Rightarrow \left(\int_S |f| du_{F^*} = 0 \right)_{\text{Mod } M_1}$$

Lemma I.3: The set of payoff configurations (x^*,F^*) are u_{F^*}-measurable.

Proof: For $S \varepsilon A(F^*)$, $u_{F^*}(S) = \int_{F^*} \chi_S du_{F^*}$. Since $\tilde{x}^* \varepsilon (x^*,F^*)$ means that $(\forall j \varepsilon F^*) x^*(j) \varepsilon M_0^+$, it suffices to consider $\sup_{j \varepsilon S} \tilde{x}^*(j) = \tilde{x}^*_S \varepsilon M_0^+$.

Then, $\left(u_{F*}(S) = 0\right)_{\text{Mod } M_1} \iff \left(\frac{\|F^* \cap S\|}{\|F^*\|} = 0\right)_{\text{Mod } M_1}$ and

$$\frac{1}{\|F^*\|} \sum_{j \in S} \tilde{x}^*(j) \leq \frac{1}{\|F^*\|} \left(\|F^* \cap S\| \cdot \tilde{x}^*_S\right).$$

However, $x^*_S \in M_0^+$ implies $\left[\left(u_{F*}(S) \cdot x^*_S\right) = 0\right]_{\text{Mod } M_1}$. Then $\left[\left(\tilde{x}^*_S \cdot \int_{F*} \chi_S du_{F*}\right) = 0\right]_{\text{Mod } M_1}$ and the result follows.

Q.E.D.

The following lemmatta will be employed.

Lemma I.4: Let $g : X \to Y^*$ for $X \subseteq R$ and Y^* internal in R^*. Then there exists an internal mapping $h : X^* \to Y^*$ such that $(h|_X) = g$.

Proof: Robinson [4], Theorem 2.11.1.

Lemma I.5: If $Q \subseteq N^*$, and Q is internal, then Q has a first element.

Proof: Robinson [4], Theorem 3.1.7.

Consider the algebra of coalitions of the game, Γ^*, $A(F^*)$, derived from the measure space, $\phi = \langle F^*, A(F^*), u_{F*}\rangle$. Then viewing $A(F^*)$ as a standardly indexed algebra of sets, u_{F*}, induces, by way of $st(u_{F*})$, a finitely additive set valued function on $A(F^*)$ with values in R_+.

The following lemma is used fundamentally in the representation scheme.

Lemma I.6: Let $\{F_j\}_{j\varepsilon N}$ be a disjoint family of sets in $A(F^*)$, then any $F_0 \subset \bigcup_{j\varepsilon N} F_j$ is such that $F_0 \subset \bigcup_{j=1}^{K} F_j$ for $K_0 \varepsilon N$. The external union of disjoint internal sets is not in $A(F^*)$.

Proof: (Loeb [6], Proposition 1)

If we consider the sequence $\{F_j\}_{j\varepsilon N}$ as a mapping from N into $A(F^*)$, then by Lemma I.4, $\{F_j\}_{j\varepsilon N}$ can be extended to the internal sequence $\{F_j\}_{j\varepsilon N^*}$.

The following set is internal, $\{K \varepsilon N^* : F_0 \subset \bigcup_{j=1}^{K} F_j\} = \tilde{K}$. Then by Lemma I.5, there is a first element to $\tilde{K}$, $\tilde{K}_0$. If $K_0 \varepsilon N^* - N$, then $F_0 \not\subset \bigcup_{j=1}^{K_0-1} F_j$. By Lemma I.5, once more, $F_0 \not\subset \bigcup_{j\varepsilon N} F_j$ by implication. This is contradictory, however. Therefore $K_0 \varepsilon N$.

Q.E.D.

Denote by $X(A^*F^*))$, the smallest σ-algebra of sets in F^*, which contains the internal algebra $A(F^*)$. Then $X(A(F^*))$ is an external algebra by the above lemma.

The principal achievement of Loeb [6] is to show, on the basis of Lemma I.6, u_{F^*} generates an outer measure on $st(u_{F^*})$ which can be extended on $X(A(F^*))$ to a standard countably additive measure space, $<F^*,X(A(F^*)),m>$. The following is a statement of Loeb's theorem.

Theorem I.7: The measure u_{F^*} extended to $st(u_{F^*})$ again extends on $<X(F^*),X(A(F^*)),m>$ to a σ-additive measure m. For $X \varepsilon X(A(F^*))$, $m(X) = \inf_{S\varepsilon A(F^*) X \supset S} st(u_{F^*}(S))$. If $st(u_{F^*}(F^*)) < \infty$

and $m(X)$ is σ-finite, then the extension is unique and for $X \in X(A(F^*))$, $m(X) = \sup_{S \in A(F^*) X \supset S} st(u_{F^*}(S))$ and for some $S \in A(F^*)$, $m((S \Delta X)) = 0$.

Proof: Loeb [6], Theorem 1, or Royden [8], Chapter 12.

Theorem I.8: Let $\Gamma^* = \langle F^*, A(F^*), v^* \rangle$ be a *Finite cooperative game defined on the *Finite measure space $\Phi^* = \langle F^*, A(F^*), u_{F^*} \rangle$ and let $\tilde{v}^*(F^*) \in M_0^+$. Then Γ^* has a standard non-atomic representation, $\psi(\Gamma^*)$ on $\psi = \langle X(F^*), X(A(F^*)), m \rangle$, the Loeb space of Φ^*.

Proof: Let each player $j \in F^*$ be mapped via the identity mapping to himself in $X(F^*)$ and denote by j_x and S_x the images of j and $S \in A(F^*)$ in $X(A(F^*))$, respectively. Since $\tilde{v}^*(F^*) \in M_0^+$ and $v^*(\cdot)$ is superadditive, $\tilde{v}^*(S) \in M_0^+$ for $S \in A(F^*)$. Let $\tilde{v}_x^*(\cdot)$ be such that $\tilde{v}_x^*(S_x) = st(\tilde{v}^*(S))$. Then $\psi(\Gamma^*) = \langle X(F^*), X(A(F^*)), \tilde{v}_x^* \rangle$ is a non-atomic representation of Γ^*.

Q.E.D.

Theorem I.9: The set of payoff configurations (x^*, F^*) for Γ^* are such that $\tilde{x}^* \in (x^*, F^*)$ implies that $st(\tilde{x}^*)$ is m-measurable in $\psi(\Gamma^*)$.

Proof: By definition, $\tilde{x}^*(j) \in M_0^+$ for all $j \in F^*$. Thus $st(\tilde{x}^*(j)) < p$ for p standard and real. In particular, $\sup_{j\in F^*} st(\tilde{x}^*(j)) < p$. By Proposition 13, p. 228, of Royden [8], it is sufficient to show that $\{j_x \in X(F^*) : st(\tilde{x}^*(j)) < p\}$ is measurable in ψ. However, $\{j_x \in X(F^*) : st(\tilde{x}^*(j)) < p\} = \bigcup_{K=1}^{\infty} \{j \in F^* : \tilde{x}^*(j) < p - 1/K\}$ for $K \in N$. The expression on the right is a member of $X(A(F^*))$.

Q.E.D.

Theorem I.10: The set of payoff configurations (x^*,F^*) for Γ^* are such that $\tilde{x}^* \in (x^*,F^*)$ implies

$$\left(\int_{X(F^*)} st(\tilde{x}^*)\,dm = \int_{F^*} \tilde{x}^*\,du_{F^*}\right)_{\text{Mod } M_1}$$

Proof: (Loeb [6], Theorem 3) Consider the strategic equivalence of Γ^* defined as $\tilde{v}^*(S) + ||S|| \cdot K$ for K standard and positive. Then (x^*,F^*) translates to $(R_+^* + K)^{F^*}$ and each $\tilde{x}^*(j) \in M_0^+ + K$ and the following set is non-empty and denumerable:

$$D = \{r \in R : m(st\tilde{x}^{*-1}(r)) > 0\}$$

Let $T = m(F^*) + 1$ and select $e > 0$ in R to be fixed. Let y_t be in R such that $\sup_{j\in F^*} st(\tilde{x}^*(j)) < y_t$ and construct the partition $0 = y_0 < y_1 < \ldots < y_t$ such that $y_i \notin D$ for $i = 1, \ldots, t$ and $y_i - y_{i-1} < e/3T$.

Allow the following to be defined:

$$\underline{S}_{u_{F*}} = \sum_{i=1}^{t} y_{i-1} u_{F*}(\tilde{x}^{*-1}[y_{i-1}, y_i))$$

$$\overline{S}_{u_{F*}} = \sum_{i=1}^{t} y_i u_{F*}(\tilde{x}^{*-1}[y_{i-1}, y_i))$$

$$\underline{S}_m = \sum_{i=1}^{t} y_{i-1} m(st\tilde{x}^{*-1}[y_{i-1}, y_i))$$

$$\overline{S}_m = \sum_{i=1}^{t} y_i m(st\tilde{x}^{*-1}[y_{i-1}, y_i))$$

Then,

$$\underline{S}_{u_{F*}} \leq \int_{F*} \tilde{x}^* du_{F*} \leq \overline{S}_{u_{F*}}$$

$$\underline{S}_m \leq \int_{X(F^*)} st(\tilde{x}^*) dm \leq \overline{S}_m$$

and

$$\overline{S}_{u_{F*}} - \underline{S}_{u_{F*}} \leq \frac{e}{3T} \sum_{i=1}^{t} u_{F*}(\tilde{x}^{*-1}[y_{i-1}, y_i)) < e/3$$

$$\overline{S}_m - \underline{S}_m \leq \frac{e}{3T} \sum_{i=1}^{t} m(st\tilde{x}^{*-1}[y_{i-1}, y_i)) < e/3$$

For i, $1 \leq i \leq t$, since $y_i \notin D$, we have that $st\tilde{x}^{*-1}(y_{i-1}, y_i) \subseteq \tilde{x}^{*-1}(y_{i-1}, y_i) \subseteq \tilde{x}^{*-1}[y_{i-1}, y_i] \subseteq st\tilde{x}^{*-1}[y_{i-1}, y_i]$. Then, $m(st\tilde{x}^{*-1}[y_{i-1}, y_i]) = m(st\tilde{x}^{*-1}[y_{i-1}, y_i)) = m(st\tilde{x}^{*-1}(y_{i-1}, y_i)) \leq m(\tilde{x}^{*-1}(y_{i-1}, y_i))$ and $u_{F*}(\tilde{x}^{*-1}(y_{i-1}, y_i)) \leq u_{F*}(\tilde{x}^{*-1}[y_{i-1}, y_i)) \leq u_{F*}\tilde{x}^{*-1}[y_{i-1} - y_i]$ and $m(\tilde{x}^{*-1}[y_{i-1}, y_i]) \leq m(st\tilde{x}^{*-1}[y_{i-1}, y_i])$.

Now $\left[m(\tilde{x}^{*-1}(y_{i-1}, y_i)) = u_{F*}(\tilde{x}^{*-1}(y_{i-1}, y_i))\right]_{\text{Mod } M_1}$ and $\left[u_{F*}\tilde{x}^{*-1}[y_{i-1}, y_i] = m(\tilde{x}^{*-1}[y_{i-1}, y_i])\right]_{\text{Mod } M_1}$ by definition of

m as $st(u_{F*})$, from which one obtains that $\left[u_{F*}(\tilde{x}^{*-1}[y_{i-1},y_i)) = m(st\tilde{x}^{*-1}[y_{i-1},y_i))\right]_{\text{Mod } M_1}$ and thus $\left[\underline{S}_{u_{F*}} = \underline{S}_m\right]_{\text{Mod } M_1}$ and finally, that

$$\left[\int_{F^*} \tilde{x}^* du_{F*} = \int_{X(F^*)} st(\tilde{x}^*)\, dm\right]_{\text{Mod } M_1}$$

Q.E.D.

Corollary I.11: For $\tilde{x}^* \varepsilon (x^*,F^*)$ for Γ^*, a nonstandard *Finite cooperative game with $\tilde{v}^*(F^*) \varepsilon M_0^+$,

$$st\left(\frac{1}{\|F^*\|} \sum_{j\varepsilon F^*} \tilde{x}^*(j)\right) = \int_{X(F^*)} st(\tilde{x}^*)\, dm$$

for $st(\tilde{x}^*) \varepsilon \left(x_X^*, X(F^*)\right)$ of $\psi(\Gamma^*)$.

Proof: Theorem I.10 and Definition I.2.

Q.E.D.

Corollary I.12: If $\tilde{x}^*$ is a payoff configuration of Γ^*, then $st(\tilde{x}^*)$ is a payoff configuration of $\psi(\Gamma^*)$, if $\tilde{v}^*(F^*) \varepsilon M_0^+$.

Proof: Since $\tilde{x}^* \varepsilon (\tilde{x}^*,F^*)$, $(\forall j \varepsilon F^*)\tilde{x}^*(j) \varepsilon (R_+^* \cap M_0^+)$, and clearly $st\tilde{x}^*(j) \varepsilon R_+$. By Corollary I.11, one has $st\left(\tilde{v}^*(F^*)\right) = st\left(\frac{1}{\|F^*\|} \sum_{j\varepsilon F^*} \tilde{x}^*(j)\right) = \int_{X(F^*)} st(\tilde{x}^*)\, dm$ and therefore $\int_{X(F^*)} st(\tilde{x}^*)\, dm = \tilde{v}_X^*(F_X^*)$.

Q.E.D.

It is now possible to characterize solution concepts for the non-atomic representations of *Finite cooperative games of the form Γ^*, based on the solution concepts developed in Part I of the series. As the characterization of other solution concepts is similar, we give that of the Quasi-Kernel as typical.

Recall from Part I that $QK^*(\Gamma^*)$ is defined as the set of payoffs in (x^*,F^*) such that $\left[\tilde{S}^*_{ij}(\tilde{x}^*) = \tilde{S}^*_{ji}(\tilde{x}^*)\right]_{\text{Mod } M_1}$ a.e. in F^*, where $\tilde{S}^*_{ij}(\tilde{x}^*) = \max_{S\varepsilon T^*_{ij}} [v^*(S) - \sum_{j\varepsilon S} \tilde{x}^*(j)]$. By Corollary I.12, the payoff configurations for $\psi(\Gamma^*)$ are the standard parts of payoff configurations for Γ^*. Then, in an analogous fashion, let $QK^*(\psi(\Gamma^*))$ be the set of payoff configurations in (x^*,F^*) for which $\left[\tilde{S}^*_{i_x j_x}(st(\tilde{x}^*)) = \tilde{S}^*_{j_x i_x}(st(\tilde{x}^*))\right]$ a.e. in $X(F^*)$, where $\tilde{S}^*_{i_x j_x}\, st(\tilde{x}^*) = \max_{S_x \varepsilon T^*_{i_x j_x}} [\tilde{v}^*_x(S_x) - \int_{S_x} st(\tilde{x}^*)dm]$ and $T^*_{i_x j_x} = \{S_x \,\varepsilon\, X(A(F^*)) : i_x \,\varepsilon\, S_x \ldots j_x \notin S_x\}$. The major difference between $QK^*(\Gamma^*)$ and $QK^*(\psi(\Gamma^*))$ is that the latter solution concept is in terms of averages and precise quality between $\tilde{S}^*_{i_x j_x}$ and $\tilde{S}^*_{j_x i_x}$ except on sets having null measure in $\psi(\Phi^*)$. The following variety of theorem is made accessible by the above characterization.

Theorem I.13: Let Γ^* be a *Finite cooperative game such that $\tilde{v}^*(F^*) \,\varepsilon\, M_0^+$, then for $\tilde{x}^* \,\varepsilon\, (\tilde{x}^*,F^*)$ $st(\tilde{x}^*) \,\varepsilon\, QK^*\psi(\Gamma^*)$ if and only if $\tilde{x}^* \varepsilon\, QK^*(\Gamma^*)$.

Proof: Let $\tilde{x}^* \varepsilon (x^*,F^*)$ such that $\tilde{x}^* \varepsilon QK^*(\Gamma^*)$. Then we wish to show that the following holds for $\psi(\Gamma^*)$:

$$\tilde{S}^*_{i_x j_x}\left(st(\tilde{x}^*)\right) = \tilde{S}^*_{j_x i_x}\left(st(\tilde{x}^*)\right) \text{ a.e. in } X(F^*)$$

If $\tilde{x}^* \varepsilon QK^*(\Gamma^*)$, then $\left[\tilde{S}^*_{ij}(\tilde{x}^*) = \tilde{S}^*_{ji}(\tilde{x}^*)\right]_{\text{Mod } M_1}$ a.e. in F^*.

Then by definition, one has

$$\left[\left|\max_{S\varepsilon T^*_{ij}} [v^*(S) - \sum_{j\varepsilon S} \tilde{x}^*(j)] - \max_{S\varepsilon T^*_{ji}} [v^*(S) - \sum_{j\varepsilon S} \tilde{x}^*(j)]\right| = 0\right]_{\text{Mod } M_1} \text{ a.e. in } F^*$$

Then

$$\left[\left|\max_{S\varepsilon T^*_{ij}} \left(\frac{1}{\|F^*\|}[v^*(S) - \sum_{j\varepsilon S} \tilde{x}^*(j)]\right) - \max_{S\varepsilon T^*_{ji}}\left(\frac{1}{\|F^*\|}[v^*(S) - \sum_{j\varepsilon S} \tilde{x}^*(j)]\right)\right| = 0\right]_{\text{Mod } M_1} \text{ a.e. in } F^*$$

which is simply,

$$\left[\left|\max_{S\varepsilon T^*_{ij}} \left(\tilde{v}^*(S) - \int_S \tilde{x}^* du_{F^*}\right) - \max_{S\varepsilon T^*_{ji}} \left(\tilde{v}^*(S) - \int_S \tilde{x}^* du_{F^*}\right)\right| = 0\right]_{\text{Mod } M_1} \text{ a.e. in } F^*$$

Then it follows that

$$st\left(\left|\max_{S\varepsilon T^*_{ij}} \left(\tilde{v}^*(S) - \int_S \tilde{x}^* du_{F^*}\right) - \max_{S\varepsilon T^*_{ji}} \left(\tilde{v}^*(S) - \int_S \tilde{x}^* du_{F^*}\right)\right|\right) = 0 \text{ a.e. in } F^*$$

which implies that

$$st\left(\max_{S\varepsilon T^*_{ij}} \left(\tilde{v}^*(S) - \int_S \tilde{x}^* du_{F^*}\right)\right) = st\left(\max_{S\varepsilon T^*_{ji}} \left(\tilde{v}^*(S) - \int_S \tilde{x}^* du_{F^*}\right)\right) \text{a.e. in } F^*$$

and, in addition, that

$$\max_{S\varepsilon T^*_{ij}} \left[st\left(\tilde{v}^*(S)\right) - st\left(\int_S \tilde{x}^* du_{F^*}\right)\right] = \max_{S\varepsilon T^*_{ji}} \left[st\left(\tilde{v}^*(S)\right) - st\left(\int_S \tilde{x}^* du_{F^*}\right)\right] \text{a.e. in } F^*$$

Since players in F^* map identically to themselves as players in $X(F^*)$, and since negligible sets in F^* are null

in X(F*), by the definitions of Theorem I.8 and the result of Corollary I.11, one obtains from the last expression that

$$\max_{S_x \varepsilon T^*_{i_x j_x}} \left[\tilde{v}^*_x(S_x) - \int_{S_x} st(\tilde{x}^*)\,dm\right] = \max_{S_x \varepsilon T^*_{j_x i_x}} \left[\tilde{v}^*_x(S_x) - \int_{S_x} st(\tilde{x}^*)\,dm\right] \text{ a.e. in } X(F^*)$$

and therefore that $\tilde{S}^*_{i_x j_x}\left(st(\tilde{x}^*)\right) = \tilde{S}^*_{j_x i_x}\left(st(\tilde{x}^*)\right)$ a.e. in X(F*). The rest is clear.

The above reasoning is completely reversible to obtain the converse.

Q.E.D.

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13. ABSTRACT

The techniques of Nonstandard Measure Theory, as developed by Professor Peter A. Loeb of the University of Illinois, are employed to demonstrate that the *Finite games treated in Part I of this series, and accordingly the solution concepts defined in that context, have standard non-atomic representations on the Loeb Space generated by the internal algebra of coalitions.

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